Backlogs in
Senior Phase Mathematics

What are they?

Where do they come from?

What can be done to address them?

7 April 2022

Craig Pournara
University of the Witwatersrand
craig.pournara@wits.ac.za

Fatima Adam
Zenex Foundation
fatima@zenexfoundation.org.za

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Our underlying assumptions

Backlogs

… are a systemic problem

… are not a result of the COVID-19 pandemic but have been exacerbated by it

… are subject-specific and phase-specific

… stem from a lack of opportunity to learn (and a lack of effort)

… can be overcome

Mathematics in the Senior Phase

… not all mathematics is equally important

… there are key **mathematical transitions** that occur in the Senior Phase

Mathematics teaching

… is part of the problem **and** part of the solution

… business-as-usual approaches won't solve the problem

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What are backlogs in SP Maths?

Backlogs are an **accumulation of gaps** in learners' **foundational** mathematical knowledge. This foundational knowledge is essential for building a **network of mathematical concepts** into which new learning can be integrated

… take time to overcome

… cannot be fixed with a clear explanation and some practice

A backlog is NOT:

… an innate deficit in a learner

… an "isolated" piece of mathematics that can be learned in a lesson or two

… a section of the curriculum that was not taught

Examples of foundational knowledge for SP maths:

Concepts: place-value, angle, length, area

Whole number properties and operations

↓ ↓

↓ Fraction properties and operations

↓ ↓

Algebraic notation and operations

A backlog develops from:

- absence of foundational knowledge
- content not/poorly taught
- lack of teacher support
- lack of learner engagement

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The key transitions in SP maths

Transition from a **numeric and calculation-based form of mathematics** to an **abstract and symbolic form of mathematics** which is more complex

Transitions …

- from whole numbers to fractions and negative numbers, and eventually to real numbers

$$8 \rightarrow \frac{1}{8} \rightarrow -8 \rightarrow \sqrt{8}$$

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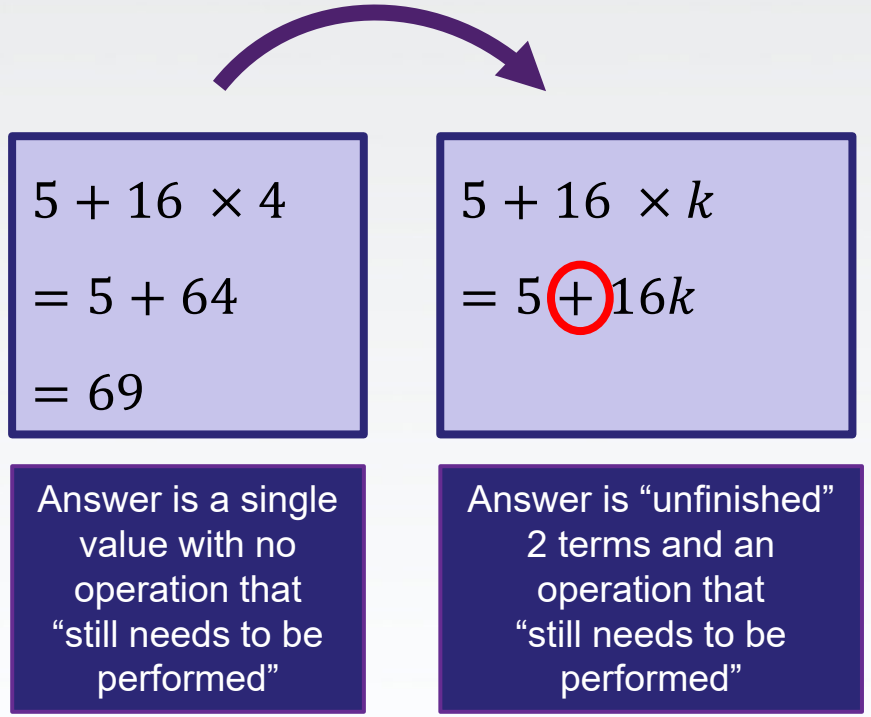
- from operating on numbers, to operating on algebraic symbols



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Transitions …

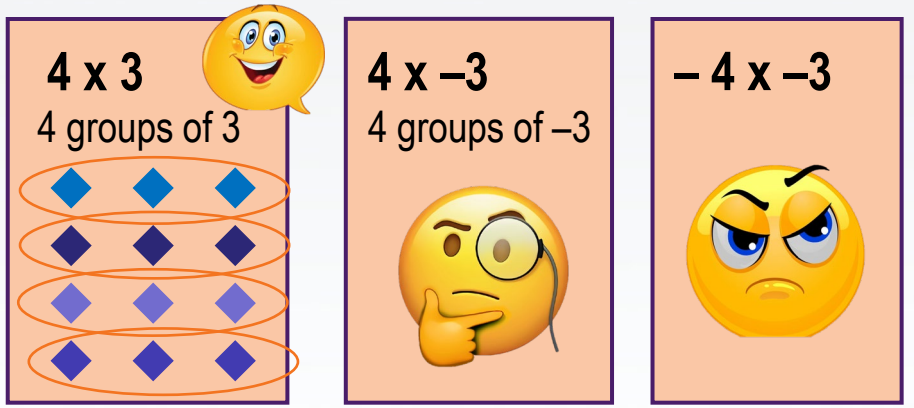
- from whole numbers to fractions and negative numbers, and eventually to real numbers
- from operating on numbers, to operating on algebraic symbols
- from using rules based on counting and numerical examples, to rules which are derived through logic and definitions

$2^5 = 2 . 2 . 2 . 2 . 2$

2 multiplied by itself 5 times

$2^{-5} =$

2 multiplied by itself **negative** 5 times ???



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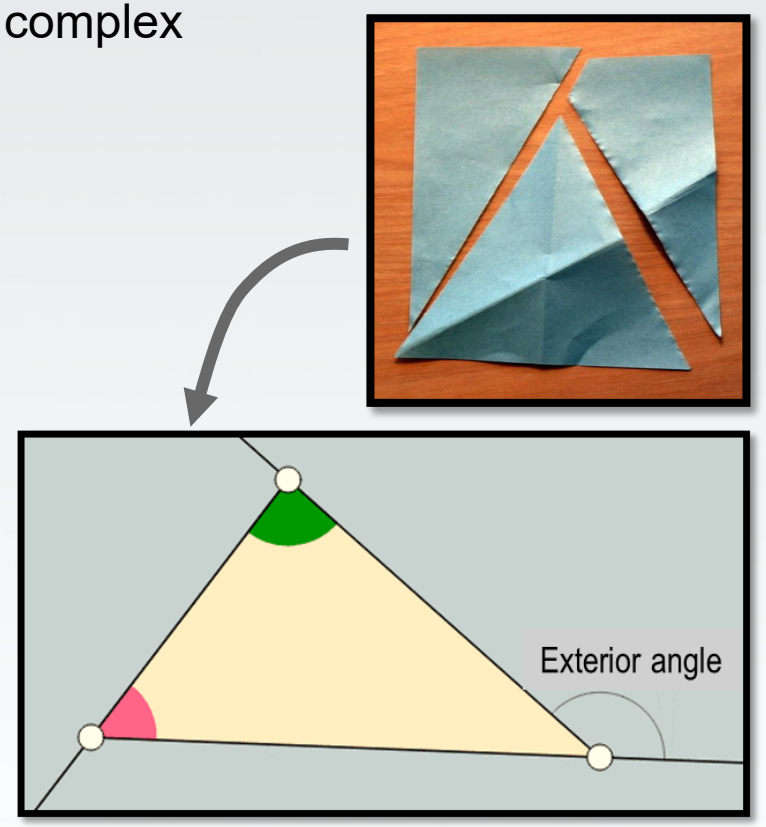
- from whole numbers to fractions and negative numbers, and eventually to real numbers
- from operating on numbers, to operating on algebraic symbols
- from using rules which are based on counting and numerical examples, to rules which are derived through logic and definitions
- from a focus on features of tangible shapes, to a focus on properties of shapes, the relationships between these properties and then logical arguments leading to geometric proofs



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If learners don't master the foundational knowledge of earlier grades and apply it fluently, they will have extreme difficulty in making the transitions required in the Senior Phase

DiBa Testing overview

Test instrument

65 multiple choice items

Grade 4 – 7 level content

Piloted in Term 4 2020

Sample

1862 Grade 8 learners who come from poor communities

11 schools: 4 township, 7 ex-Model C; 10 in GP, 1 in MP

Data collected Feb/March 2021

COVID pandemic hit hard

NO access to online learning during lockdown
Rotational learning thereafter

DiBa Test – Overall learner performance

LEARNERS

Township schools: 455

ex-Model C schools: 1407

Topic	No. of items	% correct
Whole number & operations	22	37.1
Fractions, decimals, percent	20	30.9
Measurement	8	**22.5**
Patterns, functions, algebra	10	33.9
Geometry	5	35.0
Average %		**32.7**

Selected DiBa items – Meaning of equal sign

What number must be put in the box to make the number sentence true?

$5 + 3 = \square + 7$
$5 + 3 = \square - 7$

Selected DiBa items – Meaning of equal sign

What number must be put in the box to make the number sentence true?

	Correct answer (%)	Incorrect answer = 8 (%)
$5 + 3 = \square + 7$	50	38
$5 + 3 = \square - 7$	32	33

Learners have a **do something** view of the equal sign

$5 + 3$ **gives me …**

They need to have an **equivalence** view

$5 + 3$ **is the same as …**

Without an equivalence view of the equal sign, learners will not make sense of solving equations

Selected DiBa items – Percentages

Which fraction is equivalent to 15%?

Options	$\frac{15}{20}$	$\frac{3}{20}$	$\frac{15}{10}$	$\frac{15}{1000}$

Selected DiBa items – Percentages

Which fraction is equivalent to 15%?

Options	$\frac{15}{20}$	$\frac{3}{20}$	$\frac{15}{10}$	$\frac{15}{1000}$	Blank
Responses (%)	13	**27**	27	23	10

Looking for 15:
63% chose an option with 15
More attention to numerator than denominator

Selected DiBa items – Comparing decimals

Choose the largest decimal fraction:

Options	0,354	0,0005	0,63	0,621

Selected DiBa items – Comparing decimals

Choose the largest decimal fraction:

Options	0,354	0,0005	0,63	0,621
Responses (%)	13	21	**28**	29

Selected DiBa items – Comparing decimals

Choose the largest decimal fraction:

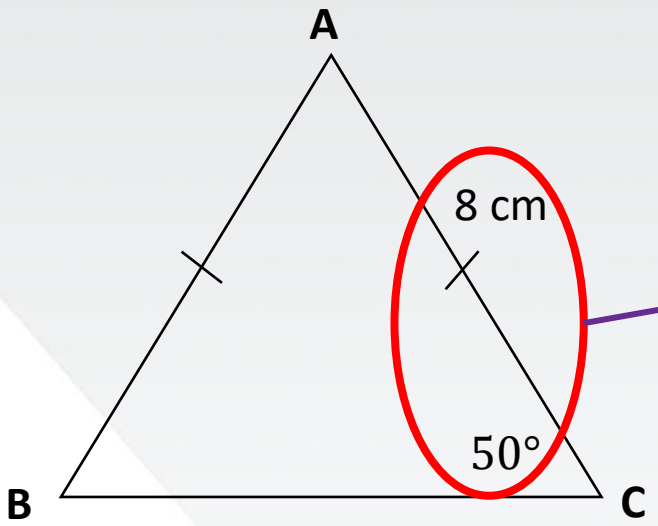
Options	0,354	0,0005	0,63	0,621	Blank
Responses (%)	13	21	**28**	29	13

Longest is largest: 0,0005 has the most decimal places

Whole number reasoning: 621 is greater than 354 and 63 and 5

Selected DiBa items - Geometry

What is the size of $\hat{B}$?



Options
8 cm
50°
58°
Impossible to tell

Selected DiBa items - Geometry

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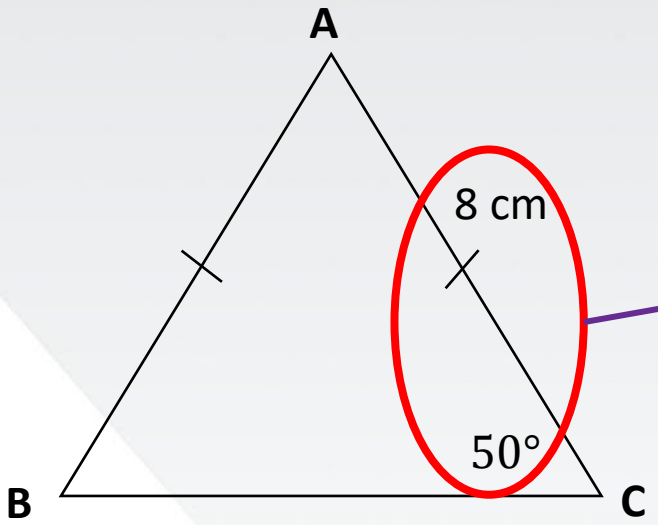
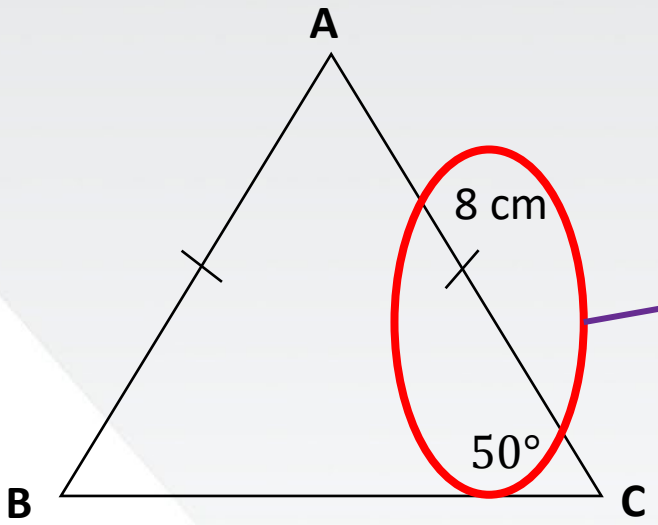


Options	Responses (%)
8 cm	16
50°	33
58°	19
Impossible to tell	8

35%

Selected DiBa items - Geometry

What is the size of $\hat{B}$?



Options	Responses (%)
8 cm	16
50°	33
58°	19
Impossible to tell	8
Blank	24

35%

Too many learners not yet distinguishing between angle-measure and length-measure

Recommendations

1) Identify **essential content** (and what is not essential)
2) Revise the SP mathematics **curriculum**
3) Dedicate **time** to deal with backlogs
4) Explore several **modalities** of interventions
5) Invite and expect increased agency and accountability from **learners**
6) Provide **teachers** with knowledge and pedagogies to address backlogs
7) Expand and strengthen **teacher support structures**

Recommendation
Revise the SP mathematics curriculum

Underlying principle: Don't introduce content briefly in lower grade if it is dealt with in detail in the next grade

Examples of possible changes

Grade	Remove	Reduce demand	Clarify scope
7	Integers Algebraic notation & simplification Statistics	Number complexity of fractions e.g. $4\frac{5}{7} - 3\frac{9}{14}$	Area, volume

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9	Quadratic equations Exponential equations	Geometric proof e.g. use of congruence for proof	Patterns Linear functions

Teach all **Statistics** in either Grade 8 or 9 (let schools decide)

Remove **Probability** entirely from SP (and GET) curriculum

Guard against **going too far** with "exotic examples" that go beyond typical use of procedure

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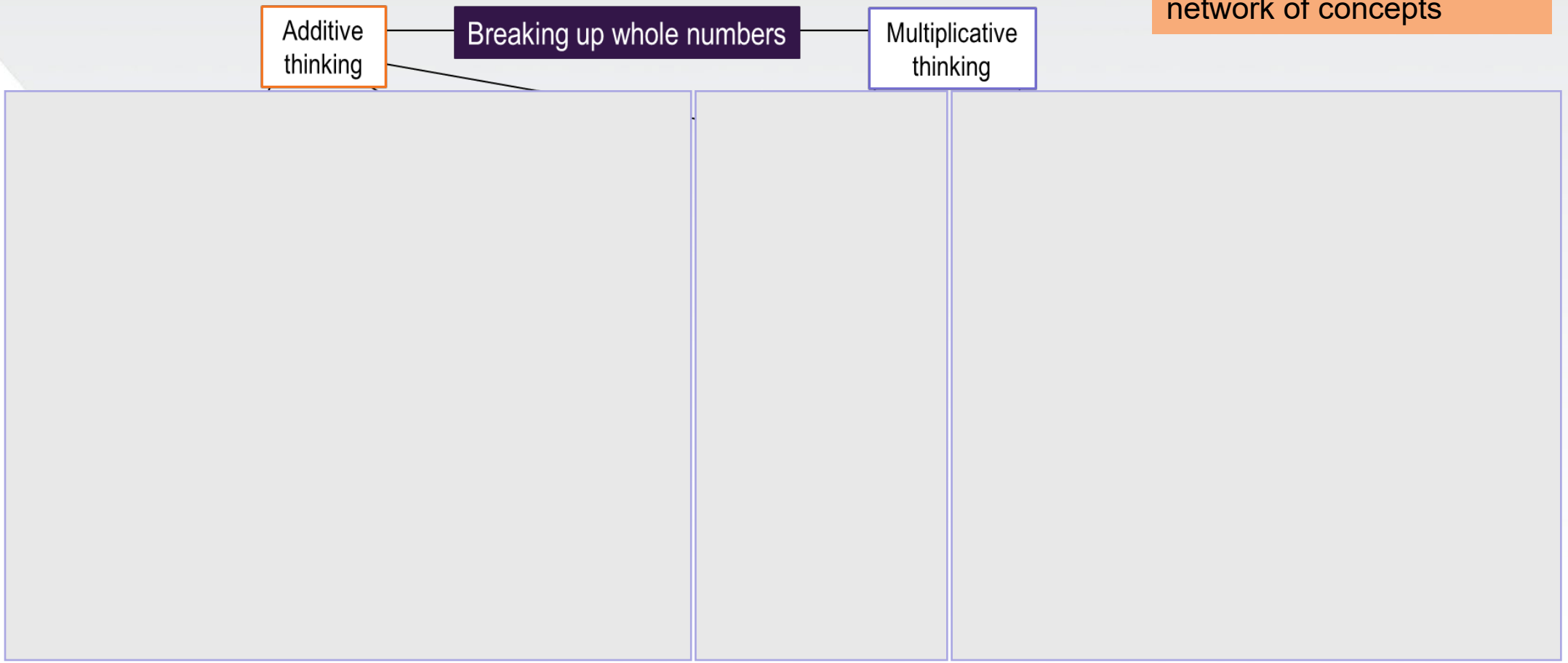
Recommendation

Provide teachers with knowledge and pedagogies to address backlogs

Underlying assumption: Teachers need to learn how to address backlogs in mathematically sound ways and with efficient strategies

Connecting pieces into a network of concepts



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Teaching with powerful representations

e.g. Empty number line

58 + 23 = 58 + 20 + 3

58 + 29 = 58 + 30 – 1

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Final thoughts

Backlogs are subject specific and phase specific

Backlogs stem from a lack of opportunity to learn, and a lack of effort and commitment

Backlogs can be overcome with time, opportunity, knowledge and commitment

Backlogs may be the single biggest challenge for mathematics teaching in SA